Autarky

The consumer preferences are expressed with a conventional utility function (U) in which every extra quantity of food or textiles consumption leads to higher utility.

Utility (U)	$U_A = C_{X,A}\, C_{Y,A}$	$U_B = C_{X,B}\, C_{Y,B}$

denominates the Consumption (C) of food (good X) in Nation A. This kind of utility function is called partially substitutable, since a reduced quantity of consumption of one of the goods can be compensated by a higher consumption of the other good. But, both goods are not fully substitutable, since the consumption of both goods must be greater than 0, otherwise utility also would be 0.

We assume labor resources (L) to be equally 100 in Nation A and Nation B.

Labor resources (L)	$L_A = 100$	$L_B = 100$

The work coefficients are as follows.

Work coefficients (W)	$W_{X,A} = 2$ $W_{Y,A} = 5$	$W_{X,B} = 5$ $W_{Y,B} = 2$

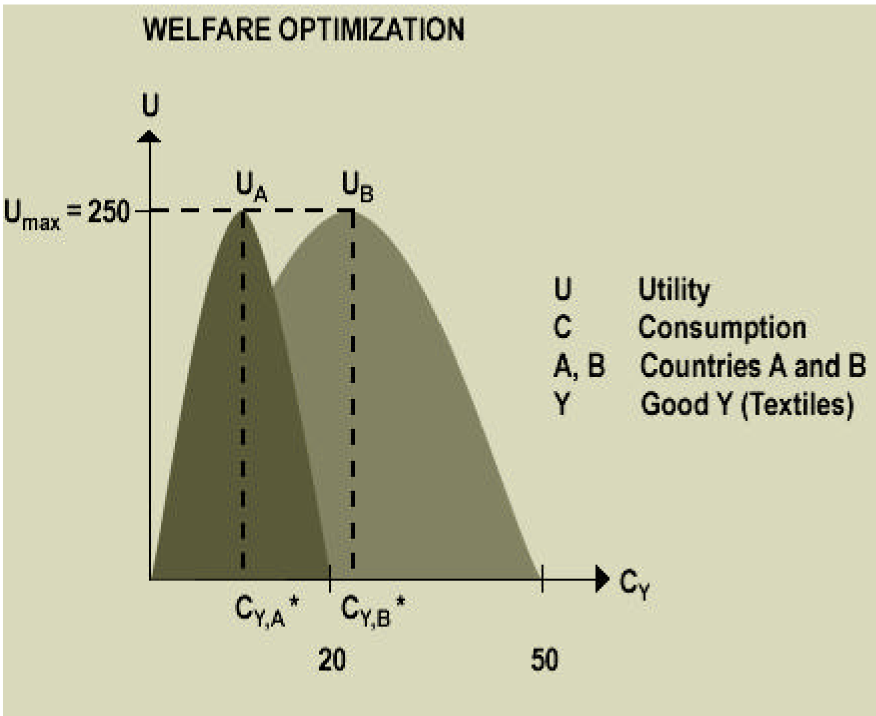
With $W_{X,A}$= 2 denominating the work coefficient of food (good X) in Nation A, which means, that in Nation A 2 labor resources are needed to produce one quantity of food. So, Nation A has a comparative advantage in producing food, whereas Nation B has its advantage in producing textiles. Both nations can use their labor force either fully for the production of one good or for any linear combination of the two.

The optimization challenge for each nation lies in maximizing utility by consuming food and textiles under the constraint of the efficient border (full and efficient use of labor resources) of their production feasibilities.

Optimization Challenge	$U_A = C_{X,A}\, C_{Y,A}$ -> max	$U_B = C_{X,B}\, C_{Y,B}$ -> max
Production Constraint	$2\, C_{X,A} + 5\, C_{Y,A} = 100$	$5\, C_{X,B} + 2\, C_{Y,B} = 100$

Inserting the production constraint into the optimization challenge leads to the transformed challenge equation.

Transformed Challenge	$U_A = (50 - 2.5\ C_{Y,A})\ C_{Y,A}$ -> max	$UB = (20 - 0.4\ C_{Y,B})\ C_{Y,B}$ -> max



The optimum of these equations is marked with □*, where the derivative equals 0.

Optimum Condition	$50 - 5\ C_{Y,A} = 0$	$20 - 0.8\ C_{Y,B} = 0$

This leads to following optimal levels of consumption and utility.

Optimum Consumption and Utility	$C_{Y,A}^* = 10$ $C_{X,A}^* = 25$ $U_A^* = 250$	$C_{Y,B}^* = 25$ $C_{X,B}^* = 10$ $U_B^* = 250$

Since there is no international trade, both nations produce both goods and consume exactly what they produce.

International Trade: Specialization, Exports, Imports and World Market Prices

In our example, the producers in Nation A specialize fully on the production of food (good X). Producers in Nation B specialize fully on textiles (good Y).

Therefore, the optimized production in each nation is as follows.

Optimized Production (P)	$P_{X,A^*} = 50$ $P_{Y,A^*} = 0$	$P_{X,B}{}^* = 0$ $P_{Y,B}{}^* = 50$

The optimization challenge is the same as under autarky.

Optimization Challenge	$U_A = (C_{X,A}\ C_{Y,A})$ -> max	$U_B = C_{X,B}\ C_{Y,B}$ -> max

With international trade, production and consumption differs by the imported and exported quantity of traded goods. As Nation A is specialized on producing food, it has to import all textiles its consumers demand. To pay these imports, it has to export a part of its food production. Therefore, it can only consume that quantity of food, which is left over after exporting.

This leads to the following cohesion.

Consumption, Imports (I) and Exports (E)	$C_{X,A} = P_{X,A}{}^* - E_{X,A}$ $C_{Y,A} = I_{Y,A}$	$C_{X,B} = I_{X,B}$ $C_{Y,B} = P_{Y,B}{}^* - E_{Y,B}$

In addition to the physical balance, the trade must also be financially balanced (in order to keep analysis simple, we abstract from the possibility of savings and loans). That means, each nation has to take in by exporting exactly that amount of money it needs to pay for its imports.

This leads to the following budget constraints of the two nations.

Budget Constraint	$p_X\ E_{X,A} = p_Y\ I_{Y,A}$	$p_X\ I_{X,B} = p_Y\ E_{Y,B}$

With p_X und p_Y standing for the price of food and textiles, respectively. As only the relative market price of the goods matters, price of food can be set to one.

Price Standardization	$pX = 1$

Inserting the budget constraint into the optimization challenge leads to the transformed challenge equation.

Transformed Challenge	$U_A = (50 - E_{X,A})\ E_{X,A} / p_Y$ -> max	$U_B = (50 - E_{Y,B})\ E_{Y,B}\ p_Y$ -> max

Again, the optimum of these equations is where the derivative equals 0.

Optimum Condition	$50 - 2\,E_{X,A} = 0$	$50 - 2\,E_{Y,B} = 0$

This leads to the following optimum quantity of exports and an import demand curve, which is dependable from (relative) world market price p_Y.

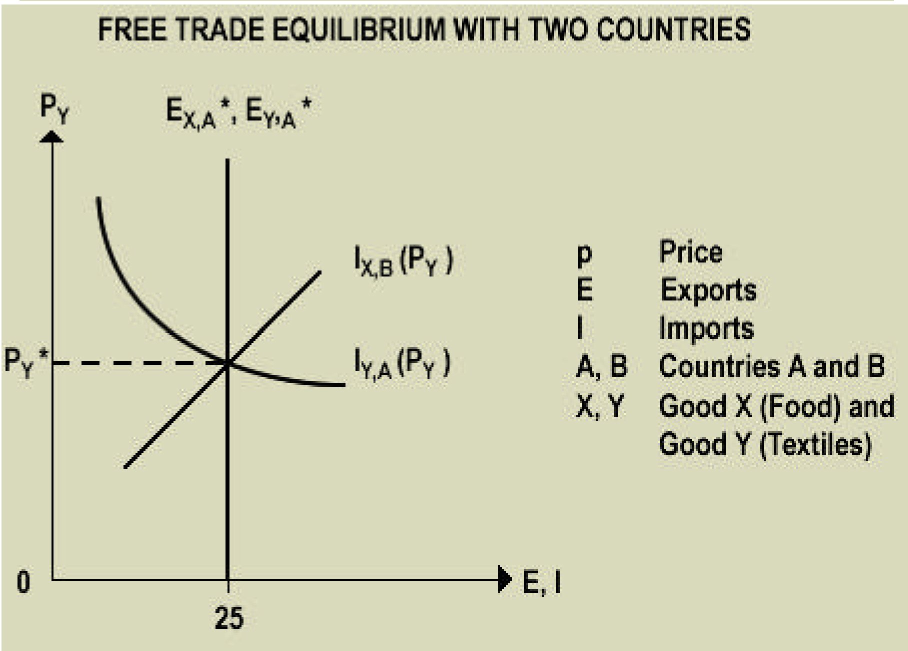
Optimum Exports	$E_{X,A}^* = 25$	$E_{Y,B}^* = 25$
Import Demand Curve	$I_{Y,A}(pY) = 25 / p_Y$	$I_{X,B} = p_Y\, 25$

Exports and imports are in equilibrium, if the quantity of the exported goods matches exactly the quantity of the imported goods.

Equilibrium	$E_{X,A}^* = I_{X,B}$ $E_{Y,B}^* = I_{Y,A}$

Inserting exports and the demand curves leads to the equilibrium price.

Transformed Equilibrium	$25 = 25 / p_Y$
Equilibrium Price	$p_Y^* = 1$



From that, we derive the optimum levels of consumption and utility.

Optimum Consumption and Utility	$C_{Y,A}^* = 25$ $C_{X,A}^* = 25$ $U_A^* = 625$	$C_{Y,B}^* = 25$ $C_{X,B}^* = 25$ $U_B^* = 625$

With international trade, both nations can improve significantly their welfare position resulting in a huge rise of their utility from 250 to 625.

Consequences of the Entrance of a Third Nation

For simplicity, let us consider Nation C, which equates Nation A. Therefore, Nation C has exactly the same optimum conditions as Nation A.

This leads to the same exports and import demand curve for Nation C as for Nation A.

Optimum Exports	$E_{X,A}^* = E_{X,C}^* = 25$	$E_{Y,B}^* = 25$
Import Demand Curve	$I_{Y,A} = I_{Y,C} = 25 / p_Y$	$I_{X,B} = p_Y\ 25$

Again, exports and imports are in equilibrium, if the quantity of the exported goods matches exactly the quantity of the imported goods. With the third nation, we have to consider, that now two nations import textiles from and export foods to Nation B.

Equilibrium	$E_{Y,B}^* = I_{Y,A} + I_{Y,C}$ $E_{X,A}^* + E_{X,C}^* = I_{X,B}$

Inserting leads to the new (relative) world market price of p_Y which is now the double of the former amount.

Transformed Equilibrium	$25 = 25 / p_Y + 25 / p_Y$
Equilibrium Price	$p_Y = 2$

The doubling of the world market price for textiles is certainly good for Nation B, but it decreases the welfare position of Nation A as the following optimum levels of consumption and utility indicate.

Optimum Consumption and Utility	$C_{Y,A}^* = C_{Y,C}^* = 12.5$ $C_{X,A}^* = C_{X,C}^* = 25$ $U_A^* = U_C^* = 312.5$	$C_{Y,B}^* = 25$ $C_{X,B}^* = 50$ $U_B^* = 1,250$

Therefore, Nation A is hit hard by extending to international trade to Nation C since its welfare reduces from 625 to 312.5 whereas Nation B can improve further its welfare from 625 to 1,250.

But, it is still better for Nation A to participate in international trade than falling back on autarky level.

Whereas Nation B is the big winner and Nation A is the big looser of the integration of Nation C into their international trades, it is possible to compensate Nation A for its welfare losses. Since Nation B improves its utility by 375 from 625 to 1,250 it can pay Nation A for its loss of 312.5 from 625 to 312.5. Even with full compensation of 312.5, Nation B would be better off than without Nation C, indicating that free international trade is indeed efficient.